The range variation of $\sigma$ for which the Laplace transform converges is called region of convergence.

**Properties of ROC of Laplace Transform**

- ROC contains strip lines parallel to $j\omega$ axis in s-plane.
- If $x(t)$ is absolutely integral and it is of finite duration, then ROC is entire s-plane.
- If $x(t)$ is a right sided sequence then ROC : $\text{Re}\{s\} > \sigma_0$.
- If $x(t)$ is a left sided sequence then ROC : $\text{Re}\{s\} < \sigma_0$.
- If $x(t)$ is a two sided sequence then ROC is the combination of two regions.

ROC can be explained by making use of examples given below:

**Example 1:** Find the Laplace transform and ROC of $x(t) = e^{-at} u(t)$

$L.T[x(t)] = L.T[e^{-at} u(t)] = \frac{1}{S+a}$

$\text{Re} > -a$

$\text{ROC} : \text{Res} >> -a$

**Example 2:** Find the Laplace transform and ROC of $x(t) = e^{at} u(-t)$

$L.T[x(t)] = L.T[e^{at} u(t)] = \frac{1}{S-a}$

$\text{Res} < a$

$\text{ROC} : \text{Res} < a$
Example 3: Find the Laplace transform and ROC of \( x(t) = e^{-at}u(t) + e^{at}u(-t) \)

\[
L.T[x(t)] = L.T[e^{-at}u(t) + e^{at}u(-t)] = \frac{1}{s+a} + \frac{1}{s-a}
\]

For \( \frac{1}{s+a} Re\{s\} > -a \)

For \( \frac{1}{s-a} Re\{s\} < a \)

Referring to the above diagram, combination region lies from \(-a\) to \(a\). Hence,

\[ ROC : -a < Re\{s\} < a \]

**Causality and Stability**

- For a system to be causal, all poles of its transfer function must be right half of s-plane.

- A system is said to be stable when all poles of its transfer function lay on the left half of s-plane.
A system is said to be unstable when at least one pole of its transfer function is shifted to the right half of s-plane.

A system is said to be marginally stable when at least one pole of its transfer function lies on the jω axis of s-plane.

ROC of Basic Functions

<table>
<thead>
<tr>
<th>ft</th>
<th>Fs</th>
<th>ROC</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u(t))</td>
<td>(\frac{1}{s})</td>
<td>ROC: Re{s} &gt; 0</td>
</tr>
<tr>
<td>(t u(t))</td>
<td>(\frac{1}{s^2})</td>
<td>ROC: Re{s} &gt; 0</td>
</tr>
<tr>
<td>(t^n u(t))</td>
<td>(\frac{n!}{s^{n+1}})</td>
<td>ROC: Re{s} &gt; 0</td>
</tr>
<tr>
<td>(e^{at} u(t))</td>
<td>(\frac{1}{s-a})</td>
<td>ROC: Re{s} &gt; a</td>
</tr>
</tbody>
</table>
\[
e^{-at} u(t) = \frac{1}{s + a}, \quad \text{ROC: Re}\{s\} > -a
\]
\[
e^{at} u(t) = -\frac{1}{s - a}, \quad \text{ROC: Re}\{s\} < a
\]
\[
e^{-at} u(-t) = -\frac{1}{s + a}, \quad \text{ROC: Re}\{s\} < -a
\]
\[
t e^{at} u(t) = \frac{1}{(s - a)^2}, \quad \text{ROC: Re}\{s\} > a
\]
\[
t^n e^{at} u(t) = \frac{n!}{(s - a)^{n+1}}, \quad \text{ROC: Re}\{s\} > a
\]
\[
t e^{-at} u(t) = \frac{1}{(s + a)^2}, \quad \text{ROC: Re}\{s\} > -a
\]
\[
t^n e^{-at} u(t) = \frac{n!}{(s + a)^{n+1}}, \quad \text{ROC: Re}\{s\} > -a
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\]
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t^n e^{-at} u(-t) = -\frac{n!}{(s + a)^{n+1}}, \quad \text{ROC: Re}\{s\} < -a
\]
\[ e^{-at} \cos bt = \frac{s + a}{(s + a)^2 + b^2} \]

\[ e^{-at} \sin bt = \frac{b}{(s + a)^2 + b^2} \]