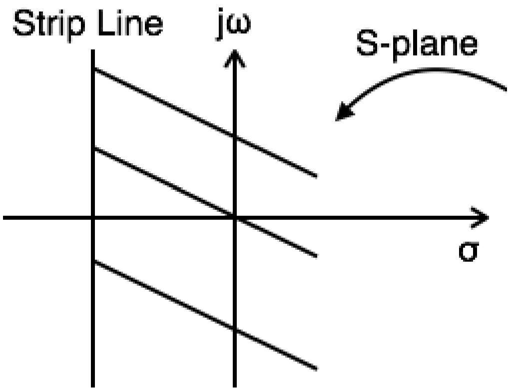
REGION OF CONVERGENCE ROC

The range variation of σ for which the Laplace transform converges is called region of convergence.

Properties of ROC of Laplace Transform

- ROC contains strip lines parallel to jω axis in s-plane.



- If xt is absolutely integral and it is of finite duration, then ROC is entire s-plane.
- If xt is a right sided sequence then ROC : Re{s} > σ_o.
- If xt is a left sided sequence then ROC : Re{s} < σ_o.
- If xt is a two sided sequence then ROC is the combination of two regions.

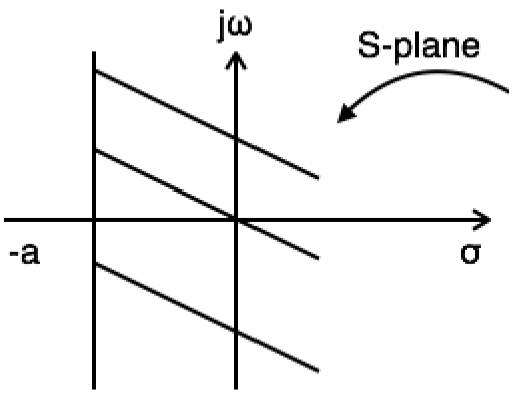
ROC can be explained by making use of examples given below:

Example 1: Find the Laplace transform and ROC of $x(t) = e\,{-^{at}}\,u(t)$

$L.T[x(t)] = L.T[e\,{-^{at}}\,u(t)] = \frac{1}{S+a}$

$Re > -a$

$ROC : Res >> -a$

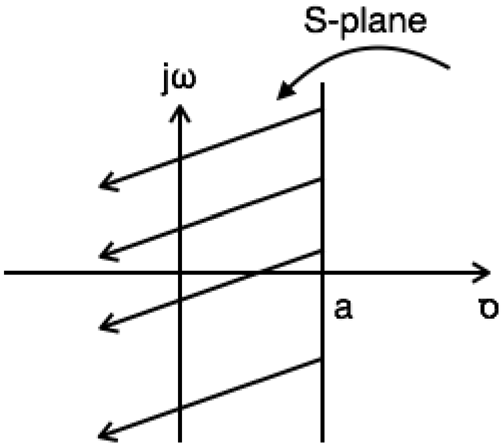


Example 2: Find the Laplace transform and ROC of $x(t) = e^{at}u(-t)$

$L.T[x(t)] = L.T[e^{at}u(t)] = \frac{1}{S-a}$

$Res < a$

$ROC : Res < a$

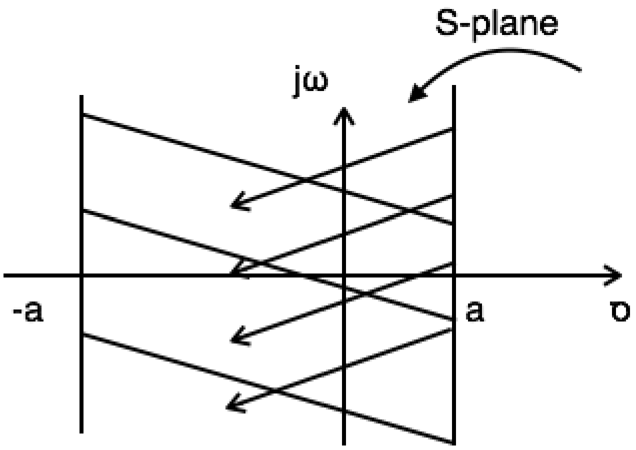


Example 3: Find the Laplace transform and ROC of $x(t) = e^{-at}u(t) + e^{at}u(-t)$

$L.T[x(t)] = L.T[e^{-at}u(t) + e^{at}u(-t)] = \frac{1}{S+a} + \frac{1}{S-a}$

For $\frac{1}{S+a} Re\{s\} > -a$

For $\frac{1}{S-a} Re\{s\} < a$

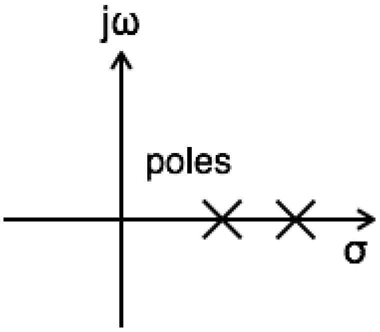


Referring to the above diagram, combination region lies from –a to a. Hence,

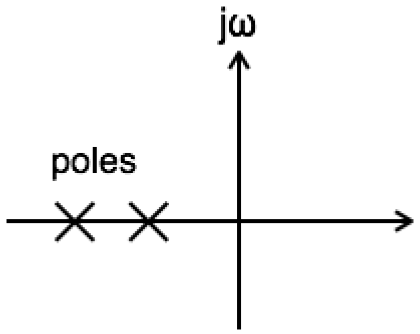
$ROC: -a < Res < a$

Causality and Stability

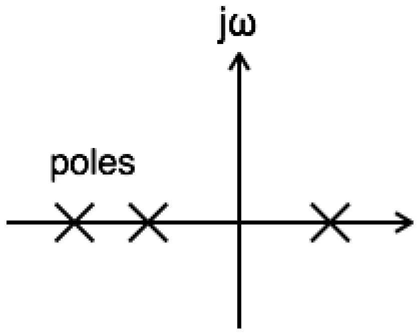
- For a system to be causal, all poles of its transfer function must be right half of s-plane.



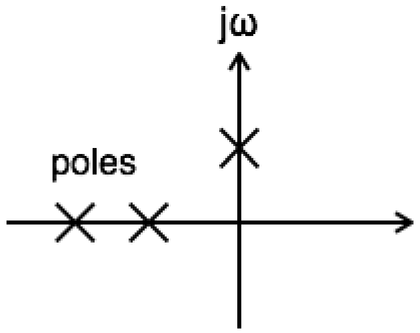
- A system is said to be stable when all poles of its transfer function lay on the left half of s-plane.



- A system is said to be unstable when at least one pole of its transfer function is shifted to the right half of s-plane.



- A system is said to be marginally stable when at least one pole of its transfer function lies on the jω axis of s-plane.



ROC of Basic Functions

ft	**F**s	**ROC**
$u(t)$	$\frac{1}{s}$	ROC: Re{s} > 0
$t\,u(t)$	$\frac{1}{s^2}$	ROC:Re{s} > 0
$t^n\,u(t)$	$\frac{n!}{s^{n+1}}$	ROC:Re{s} > 0
$e^{at}\,u(t)$	$\frac{1}{s-a}$	ROC:Re{s} > a

$e^{-at}\,u(t)$	$\frac{1}{s+a}$	ROC:Re{s} > -a
$e^{at}\,u(t)$	$-\frac{1}{s-a}$	ROC:Re{s} < a
$e^{-at}\,u(-t)$	$-\frac{1}{s+a}$	ROC:Re{s} < -a
$t\,e^{at}\,u(t)$	$\frac{1}{(s-a)^2}$	ROC:Re{s} > a
$t^n e^{at}\,u(t)$	$\frac{n!}{(s-a)^{n+1}}$	ROC:Re{s} > a
$t\,e^{-at}\,u(t)$	$\frac{1}{(s+a)^2}$	ROC:Re{s} > -a
$t^n\,e^{-at}\,u(t)$	$\frac{n!}{(s+a)^{n+1}}$	ROC:Re{s} > -a
$t\,e^{at}\,u(-t)$	$-\frac{1}{(s-a)^2}$	ROC:Re{s} < a
$t^n\,e^{at}\,u(-t)$	$-\frac{n!}{(s-a)^{n+1}}$	ROC:Re{s} < a
$t\,e^{-at}\,u(-t)$	$-\frac{1}{(s+a)^2}$	ROC:Re{s} < -a
$t^n\,e^{-at}\,u(-t)$	$-\frac{n!}{(s+a)^{n+1}}$	ROC:Re{s} < -a

$e^{-at} \cos bt$	$\frac{s+a}{(s+a)^2 + b^2}$
$e^{-at} \sin bt$	$\frac{b}{(s+a)^2 + b^2}$