MATLAB - INTEGRATION

Integration deals with two essentially different types of problems.

- In the first type, derivative of a function is given and we want to find the function. Therefore, we basically reverse the process of differentiation. This reverse process is known as anti-differentiation, or finding the primitive function, or finding an **indefinite integral**.
- The second type of problems involve adding up a very large number of very small quantities and then taking a limit as the size of the quantities approaches zero, while the number of terms tend to infinity. This process leads to the definition of the **definite integral**.

Definite integrals are used for finding area, volume, center of gravity, moment of inertia, work done by a force, and in numerous other applications.

Finding Indefinite Integral Using MATLAB

By definition, if the derivative of a function fx is f'x, then we say that an indefinite integral of f'x with respect to x is fx. For example, since the derivative *withrespecttox* of x^2 is 2x, we can say that an indefinite integral of 2x is x^2.

In symbols −

$\mathbf{f'(x^2) = 2x}$, therefore,

$\mathbf{\int 2xdx = x^2.}$

Indefinite integral is not unique, because derivative of $x^2 + c$, for any value of a constant c, will also be 2x.

This is expressed in symbols as −

$\mathbf{\int 2xdx = x^2 + c}$.

Where, c is called an 'arbitrary constant'.

MATLAB provides an **int** command for calculating integral of an expression. To derive an expression for the indefinite integral of a function, we write −

```
int(f);
```

For example, from our previous example −

```
syms x
int(2*x)
```

MATLAB executes the above statement and returns the following result −

```
ans =
 x^2
```

Example 1

In this example, let us find the integral of some commonly used expressions. Create a script file and type the following code in it −

```
syms x n
int(sym(x^n))
f = 'sin(n*t)'
int(sym(f))
```

```
syms a t
int(a*cos(pi*t))
int(a^x)
```

When you run the file, it displays the following result −

```
ans =
 piecewise([n == -1, log(x)], [n ~= -1, x^(n + 1)/(n + 1)])
f =
sin(n*t)
ans =
 -cos(n*t)/n
 ans =
 (a*sin(pi*t))/pi
 ans =
 a^x/log(a)
```

Example 2

Create a script file and type the following code in it −

```
syms x n
int(cos(x))
int(exp(x))
int(log(x))
int(x^-1)
int(x^5*cos(5*x))
pretty(int(x^5*cos(5*x)))
int(x^-5)
int(sec(x)^2)
pretty(int(1 - 10*x + 9 * x^2))
int((3 + 5*x -6*x^2 - 7*x^3)/2*x^2)
pretty(int((3 + 5*x -6*x^2 - 7*x^3)/2*x^2))
```

Note that the **pretty** function returns an expression in a more readable format.

When you run the file, it displays the following result −

```
ans =

sin(x)


ans =

exp(x)


ans =

x*(log(x) - 1)


ans =

log(x)


ans =

(24*cos(5*x))/3125 + (24*x*sin(5*x))/625 - (12*x^2*cos(5*x))/125 + (x^4*cos(5*x))/5 -
(4*x^3*sin(5*x))/25 + (x^5*sin(5*x))/5


                                    2             4
  24 cos(5 x)   24 x sin(5 x)   12 x  cos(5 x)   x  cos(5 x)
  ----------- + ------------- - -------------- + ----------- -
```

```
     3125              625              125               5

        3             5

    4 x   sin(5 x)    x   sin(5 x)
     ------------- + -----------
          25               5

ans =

-1/(4*x^4)


ans =

tan(x)


        2
  x (3 x   - 5 x + 1)

ans =

- (7*x^6)/12 - (3*x^5)/5 + (5*x^4)/8 + x^3/2


       6      5      4    3
    7 x    3 x    5 x    x
  - ---- - ---- + ---- + --
     12     5      8     2
```

Finding Definite Integral Using MATLAB

By definition, definite integral is basically the limit of a sum. We use definite integrals to find areas such as the area between a curve and the x-axis and the area between two curves. Definite integrals can also be used in other situations, where the quantity required can be expressed as the limit of a sum.

The **int** function can be used for definite integration by passing the limits over which you want to calculate the integral.

To calculate

$$\int_a^b f(x)dx = f(b) - f(a)$$

we write,

```
int(x, a, b)
```

For example, to calculate the value of

$$\int_4^9 x dx$$

we write −

```
int(x, 4, 9)
```

MATLAB executes the above statement and returns the following result −

```
ans =
 65/2
```

Following is Octave equivalent of the above calculation −

```
pkg load symbolic
symbols

x = sym("x");

f = x;

c = [1, 0];
integral = polyint(c);

a = polyval(integral, 9) - polyval(integral, 4);

display('Area: '), disp(double(a));
```

Octave executes the code and returns the following result −

```
Area:

 32.500
```

An alternative solution can be given using quad function provided by Octave as follows −

```
pkg load symbolic
symbols

f = inline("x");
[a, ierror, nfneval] = quad(f, 4, 9);

display('Area: '), disp(double(a));
```

Octave executes the code and returns the following result −

```
Area:

 32.500
```

Example 1

Let us calculate the area enclosed between the x-axis, and the curve $y = x^3 - 2x + 5$ and the ordinates x = 1 and x = 2.

The required area is given by −

$$A = \int_1^2 (x^3 - 2x + 5) dx$$

Create a script file and type the following code −

```
f = x^3 - 2*x +5;
a = int(f, 1, 2)
display('Area: '), disp(double(a));
```

When you run the file, it displays the following result −

```
a =
23/4
Area:
    5.7500
```

Following is Octave equivalent of the above calculation −

```
pkg load symbolic
```

```
symbols

x = sym("x");

f = x^3 - 2*x +5;

c = [1, 0, -2, 5];
integral = polyint(c);

a = polyval(integral, 2) - polyval(integral, 1);

display('Area: '), disp(double(a));
```

Octave executes the code and returns the following result –

```
Area:

 5.7500
```

An alternative solution can be given using quad function provided by Octave as follows –

```
pkg load symbolic
symbols

x = sym("x");

f = inline("x^3 - 2*x +5");

[a, ierror, nfneval] = quad(f, 1, 2);
display('Area: '), disp(double(a));
```

Octave executes the code and returns the following result –

```
Area:

 5.7500
```

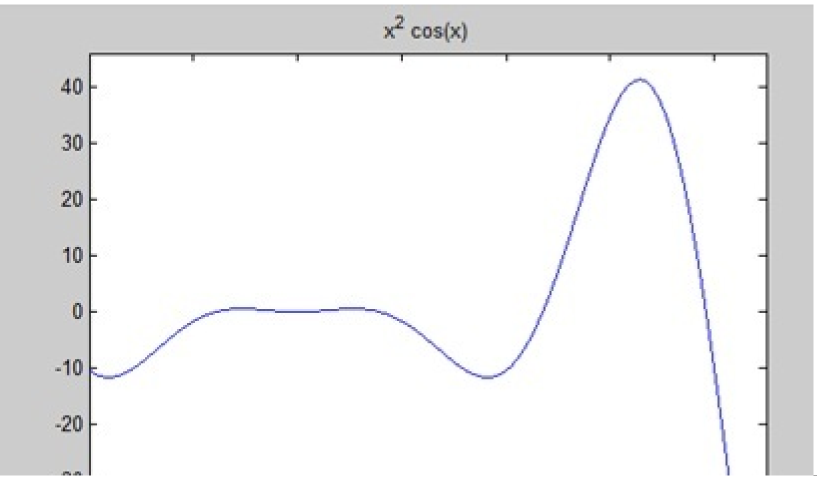
Example 2

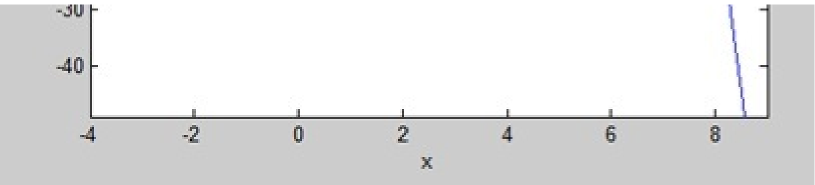
Find the area under the curve: $f_x = x^2 \cos x$ for $-4 \le x \le 9$.

Create a script file and write the following code –

```
f = x^2*cos(x);
ezplot(f, [-4,9])
a = int(f, -4, 9)
disp('Area: '), disp(double(a));
```

When you run the file, MATLAB plots the graph –





The output is given below −

```
a =

8*cos(4) + 18*cos(9) + 14*sin(4) + 79*sin(9)

Area:
    0.3326
```

Following is Octave equivalent of the above calculation −

```
pkg load symbolic
symbols

x = sym("x");

f = inline("x^2*cos(x)");

ezplot(f, [-4,9])
print -deps graph.eps

[a, ierror, nfneval] = quad(f, -4, 9);

display('Area: '), disp(double(a));
```