



GRAPH THEORY

tutorialspoint
SIMPLY EASY LEARNING

www.tutorialspoint.com

 <https://www.facebook.com/tutorialspointindia>

 <https://twitter.com/tutorialspoint>

About the Tutorial

This tutorial offers a brief introduction to the fundamentals of graph theory. Written in a reader-friendly style, it covers the types of graphs, their properties, trees, graph traversability, and the concepts of coverings, coloring, and matching.

Audience

This tutorial has been designed for students who want to learn the basics of Graph Theory. Graph Theory has a wide range of applications in engineering and hence, this tutorial will be quite useful for readers who are into Language Processing or Computer Networks, physical sciences and numerous other fields.

Prerequisites

Before you start with this tutorial, you need to know elementary number theory and basic set operations in Mathematics. It is mandatory to have a basic knowledge of Computer Science as well.

Disclaimer & Copyright

© Copyright 2015 by Tutorials Point (I) Pvt. Ltd.

All the content and graphics published in this e-book are the property of Tutorials Point (I) Pvt. Ltd. The user of this e-book is prohibited to reuse, retain, copy, distribute or republish any contents or a part of contents of this e-book in any manner without written consent of the publisher. We strive to update the contents of our website and tutorials as timely and as precisely as possible, however, the contents may contain inaccuracies or errors. Tutorials Point (I) Pvt. Ltd. provides no guarantee regarding the accuracy, timeliness or completeness of our website or its contents including this tutorial. If you discover any errors on our website or in this tutorial, please notify us at contact@tutorialspoint.com.

Table of Contents

	About the Tutorial	i
	Audience	i
	Prerequisites	i
	Disclaimer & Copyright	i
	Table of Contents	ii
1.	GRAPH THEORY – INTRODUCTION	1
	What is a Graph?	1
	Applications of Graph Theory	1
2.	GRAPH THEORY – FUNDAMENTALS	3
	Point.....	3
	Line.....	3
	Vertex.....	3
	Edge	4
	Graph	4
	Loop	5
	Degree of Vertex	6
	Degree of Vertex in an Undirected Graph	6
	Degree of Vertex in a Directed Graph	7
	Pendent Vertex	10
	Isolated Vertex	10
	Adjacency	10
	Parallel Edges	12
	Multi Graph	12
	Degree Sequence of a Graph	13

3.	GRAPH – BASIC PROPERTIES	15
	Distance between Two Vertices.....	15
	Eccentricity of a Vertex.....	16
	Radius of a Connected Graph	16
	Diameter of a Graph.....	17
	Central Point	17
	Centre.....	17
	Circumference	17
	Girth	18
	Sum of Degrees of Vertices Theorem.....	18
4.	TYPES OF GRAPHS.....	19
	Null Graph	19
	Trivial Graph.....	19
	Non-Directed Graph	20
	Directed Graph	20
	Simple Graph.....	21
	Connected Graph.....	23
	Disconnected Graph	23
	Regular Graph	24
	Complete Graph	25
	Cycle Graph	26
	Wheel Graph	27
	Cyclic Graph.....	28
	Acyclic Graph.....	28
	Bipartite Graph.....	28
	Complete Bipartite Graph.....	29
	Star Graph	30

	Complement of a Graph	31
5.	TREES.....	33
	Tree	33
	Forest	34
	Spanning Trees	34
	Circuit Rank	35
	Kirchoff's Theorem	36
6.	CONNECTIVITY	38
	Connectivity	38
	Cut Vertex	39
	Cut Edge (Bridge)	40
	Cut Set of a Graph	42
	Edge Connectivity	43
	Vertex Connectivity	44
7.	COVERINGS.....	46
	Line Covering.....	46
	Minimal Line Covering	47
	Minimum Line Covering.....	47
	Vertex Covering.....	48
	Minimal Vertex Covering	48
	Minimum Vertex Covering.....	49
8.	MATCHINGS.....	50
	Matching	50
	Maximal Matching.....	51
	Maximum Matching	52

	Perfect Matching	53
9.	INDEPENDENT SETS	55
	Independent Line Set.....	55
	Maximal Independent Line Set	56
	Maximum Independent Line Set.....	56
	Independent Vertex Set.....	58
	Maximal Independent Vertex Set	58
	Maximum Independent Vertex Set.....	60
10.	COLORING.....	62
	Vertex Coloring.....	62
	Chromatic Number	62
	Region Coloring	63
	Applications of Graph Coloring.....	64
11.	ISOMORPHISM.....	65
	Isomorphic Graphs	65
	Planar Graphs.....	67
	Regions.....	67
	Homomorphism	71
12.	TRAVERSABILITY.....	73
	Euler's Path	73
	Euler's Circuit	74
	Euler's Circuit Theorem	74
	Hamiltonian Graph	75
	Hamiltonian Path.....	75
13.	EXAMPLES.....	77
	Example 1.....	77

Example 2	78
Example 3	78
Example 4	79
Example 5	79
Example 6	80

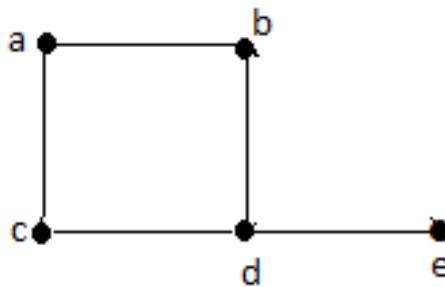
1. GRAPH THEORY – INTRODUCTION

In the domain of mathematics and computer science, *graph theory is the study of graphs that concerns with the relationship among edges and vertices*. It is a popular subject having its applications in computer science, information technology, biosciences, mathematics, and linguistics to name a few. Without further ado, let us start with defining a graph.

What is a Graph?

A graph is a pictorial representation of a set of objects where some pairs of objects are connected by links. The interconnected objects are represented by points termed as **vertices**, and the links that connect the vertices are called **edges**.

Formally, a graph is a pair of sets **(V, E)**, where **V** is the set of vertices and **E** is the set of edges, connecting the pairs of vertices. Take a look at the following graph:



In the above graph,

$$V = \{a, b, c, d, e\}$$

$$E = \{ab, ac, bd, cd, de\}$$

Applications of Graph Theory

Graph theory has its applications in diverse fields of engineering:

- **Electrical Engineering** – The concepts of graph theory is used extensively in designing circuit connections. The types or organization of connections are named as topologies. Some examples for topologies are star, bridge, series, and parallel topologies.
- **Computer Science** – Graph theory is used for the study of algorithms. For example,

- Kruskal's Algorithm
- Prim's Algorithm
- Dijkstra's Algorithm

- **Computer Network** – The relationships among interconnected computers in the network follows the principles of graph theory.

- **Science** – The molecular structure and chemical structure of a substance, the DNA structure of an organism, etc., are represented by graphs.

- **Linguistics** – The parsing tree of a language and grammar of a language uses graphs.

- **General** – Routes between the cities can be represented using graphs. Depicting hierarchical ordered information such as family tree can be used as a special type of graph called tree.

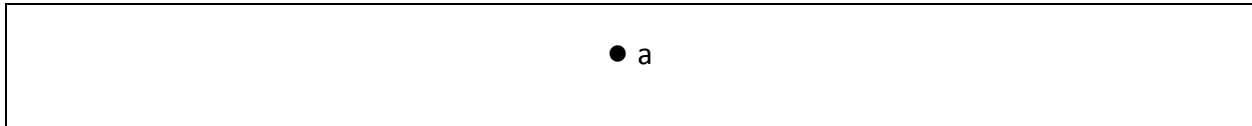
2. GRAPH THEORY – FUNDAMENTALS

A graph is a diagram of points and lines connected to the points. It has at least one line joining a set of two vertices with no vertex connecting itself. The concept of graphs in graph theory stands up on some basic terms such as point, line, vertex, edge, degree of vertices, properties of graphs, etc. Here, in this chapter, we will cover these fundamentals of graph theory.

Point

A **point** is a particular position in a one-dimensional, two-dimensional, or three-dimensional space. For better understanding, a point can be denoted by an alphabet. It can be represented with a dot.

Example

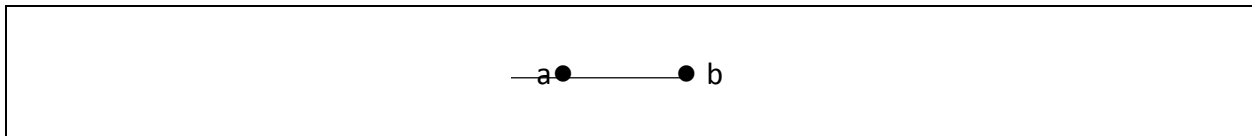


Here, the dot is a point named 'a'.

Line

A **Line** is a connection between two points. It can be represented with a solid line.

Example

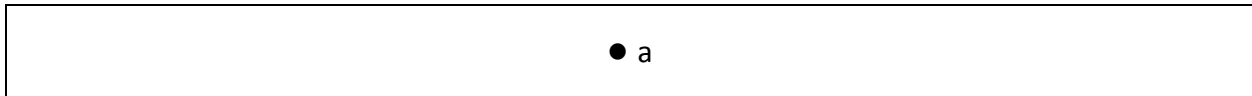


Here, 'a' and 'b' are the points. The link between these two points is called a line.

Vertex

A vertex is a point where multiple lines meet. It is also called a **node**. Similar to points, a vertex is also denoted by an alphabet.

Example

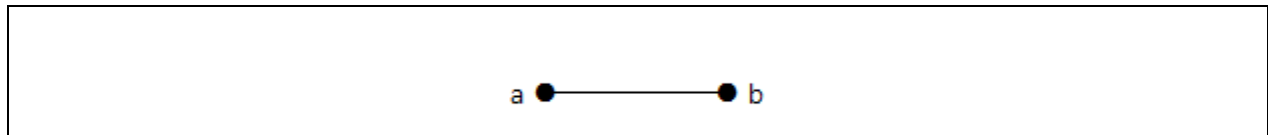


Here, the vertex is named with an alphabet 'a'.

Edge

An edge is the mathematical term for a line that connects two vertices. Many edges can be formed from a single vertex. Without a vertex, an edge cannot be formed. There must be a starting vertex and an ending vertex for an edge.

Example

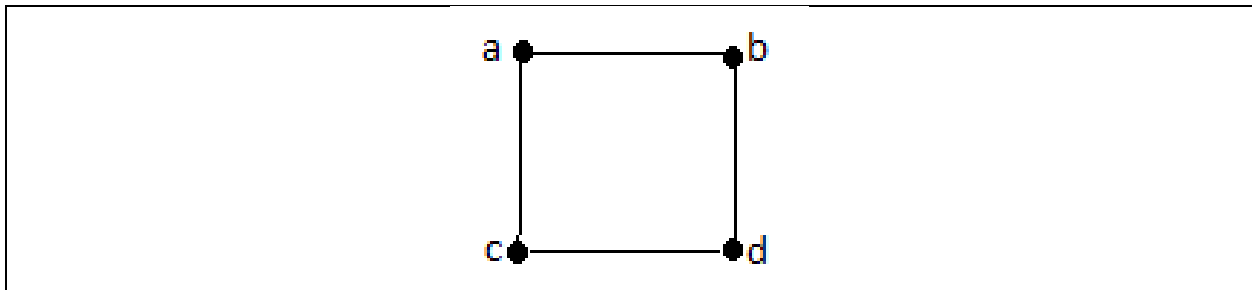


Here, 'a' and 'b' are the two vertices and the link between them is called an edge.

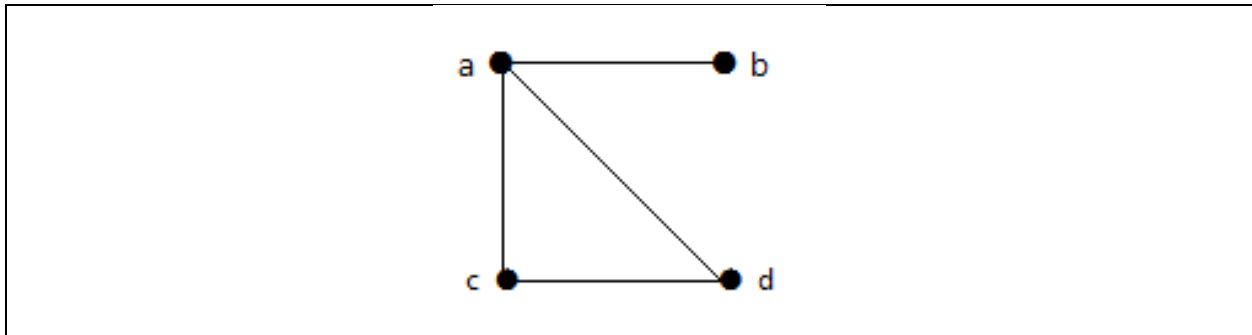
Graph

A graph 'G' is defined as $G = (V, E)$ Where V is a set of all vertices and E is a set of all edges in the graph.

Example 1



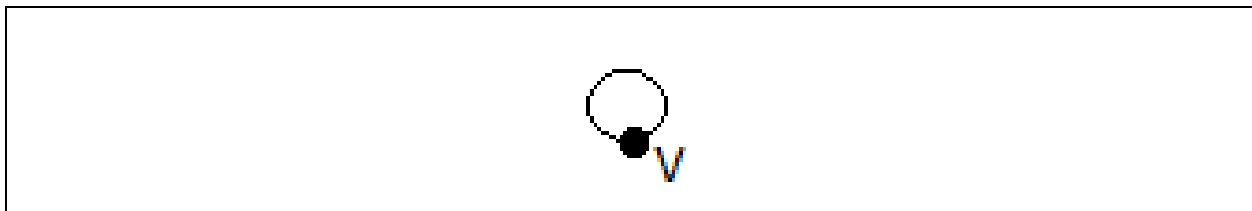
In the above example, ab, ac, cd, and bd are the edges of the graph. Similarly, a, b, c, and d are the vertices of the graph.

Example 2

In this graph, there are four vertices a , b , c , and d , and four edges ab , ac , ad , and cd .

Loop

In a graph, if an edge is drawn from vertex to itself, it is called a loop.

Example 1

In the above graph, V is a vertex for which it has an edge (V, V) forming a loop.

Example 2

In this graph, there are two loops which are formed at vertex a , and vertex b .

Degree of Vertex

It is the number of vertices incident with the vertex V .

Notation: $\deg(V)$.

In a simple graph with n number of vertices, the degree of any vertices is:

$$\deg(v) \leq n - 1 \quad \forall \quad v \in G$$

A vertex can form an edge with all other vertices except by itself. So the degree of a vertex will be up to the **number of vertices in the graph minus 1**. This 1 is for the self-vertex as it cannot form a loop by itself. If there is a loop at any of the vertices, then it is not a Simple Graph.

Degree of vertex can be considered under two cases of graphs:

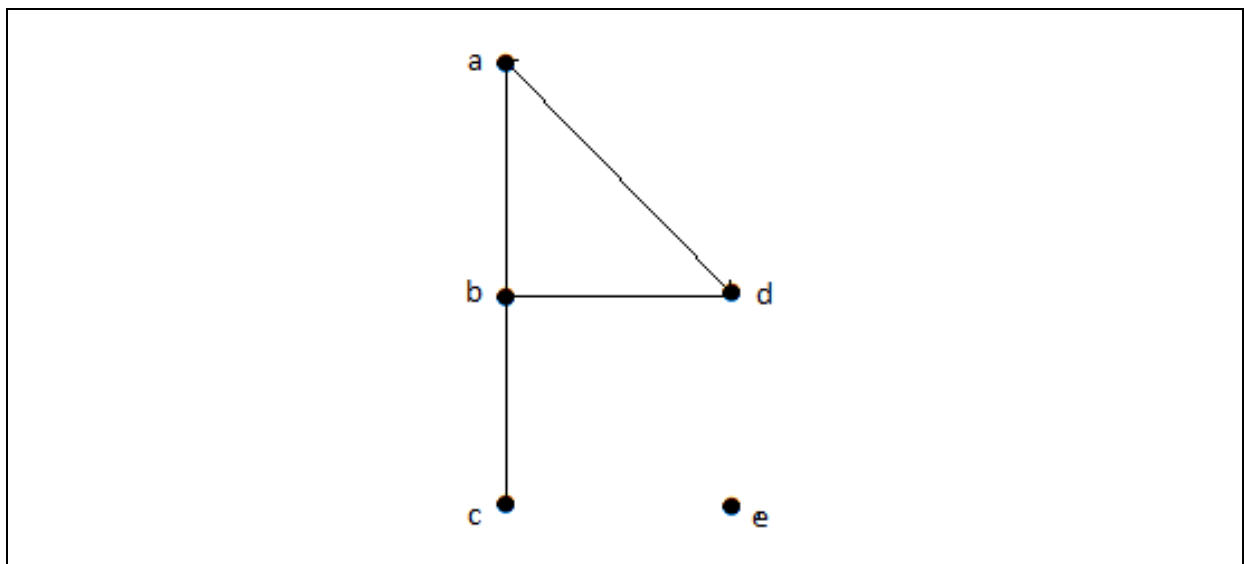
- Undirected Graph
- Directed Graph

Degree of Vertex in an Undirected Graph

An undirected graph has no directed edges. Consider the following examples.

Example 1

Take a look at the following graph:



In the above Undirected Graph,

- $\text{deg}(a) = 2$, as there are 2 edges meeting at vertex 'a'.
- $\text{deg}(b) = 3$, as there are 3 edges meeting at vertex 'b'.
- $\text{deg}(c) = 1$, as there is 1 edge formed at vertex 'c'

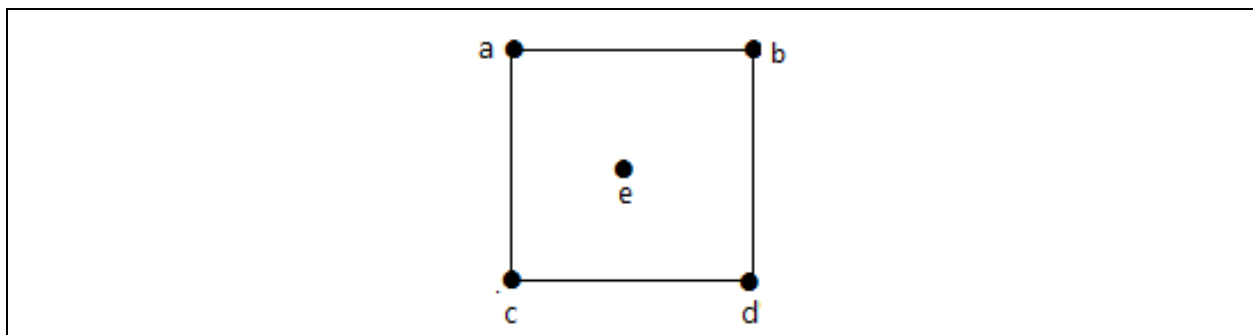
So 'c' is a **pendent vertex**.

- $\text{deg}(d) = 2$, as there are 2 edges meeting at vertex 'd'.
- $\text{deg}(e) = 0$, as there are 0 edges formed at vertex 'e'.

So 'e' is an **isolated vertex**.

Example 2

Take a look at the following graph:



In the above graph,

$$\text{deg}(a) = 2, \text{deg}(b) = 2, \text{deg}(c) = 2, \text{deg}(d) = 2, \text{and } \text{deg}(e) = 0.$$

The vertex 'e' is an isolated vertex. The graph does not have any pendent vertex.

Degree of Vertex in a Directed Graph

In a directed graph, each vertex has an **indegree** and an **outdegree**.

Indegree of a Graph

- Indegree of vertex V is the number of edges which are coming into the vertex V .
- **Notation:** $\text{deg}^+(V)$.

End of ebook preview
If you liked what you saw...
Buy it from our store @ [**https://store.tutorialspoint.com**](https://store.tutorialspoint.com)