

BOOLEAN EXPRESSION / FUNCTION

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Boolean algebra deals with binary variables and logic operation. A **Boolean Function** is described by an algebraic expression called **Boolean expression** which consists of binary variables, the constants 0 and 1, and the logic operation symbols. Consider the following example.

$$\begin{array}{lcl} F(A, B, C, D) & = & A + \overline{BC} + ADC \\ \text{Boolean Function} & & \text{Boolean Expression} \end{array} \quad \text{Equation No. 1}$$

Here the left side of the equation represents the output Y. So we can state equation no. 1

$$Y = A + \overline{BC} + ADC$$

Truth Table Formation

A truth table represents a table having all combinations of inputs and their corresponding result.

It is possible to convert the switching equation into a truth table. For example, consider the following switching equation.

$$F(A, B, C) = A + BC$$

The output will be high 1 if $A = 1$ or $BC = 1$ or both are 1. The truth table for this equation is shown by Table a. The number of rows in the truth table is 2^n where n is the number of input variables $n = 3$ for the given equation. Hence there are $2^3 = 8$ possible input combination of inputs.

Inputs			Output
A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Methods to simplify the boolean function

The methods used for simplifying the Boolean function are as follows –

- Karnaugh-map or K-map, and
- NAND gate method.

Karnaugh-map or K-map

The Boolean theorems and the De-Morgan's theorems are useful in manipulating the logic expression. We can realize the logical expression using gates. The number of logic gates required for the realization of a logical expression should be reduced to a minimum possible value by K-map method. This method can be done in two different ways, as discussed below.

Sum of Products SOP Form

It is in the form of sum of three terms AB, AC, BC with each individual term is a product of two variables. Say A.B or A.C etc. Therefore such expressions are known as expression in SOP form. The sum and products in SOP form are not the actual additions or multiplications. In fact they are the OR and AND functions. In SOP form, 0 represents a bar and 1 represents an unbar. SOP form is represented by



Given below is an example of SOP.

In SOP form $\overline{A}\overline{B} + \overline{A}B + AB$

$\downarrow \downarrow$ $\downarrow \downarrow$ $\downarrow \downarrow$
 00 01 11

A \ B	0	1
0	1	1
1		1

$\sum m(0,1,3)$

Answer: $\overline{A}\overline{B} + \overline{A}B + AB = \overline{A} + B$

Product of Sums POS Form

It is in the form of product of three terms $A+B$, $B+C$, or $A+C$ with each term is in the form of a sum of two variables. Such expressions are said to be in the product of sums POS form. In POS form, 0 represents an unbar and 1 represents a bar. POS form is represented by



Given below is an example of POS.

In POS form $(B + \overline{C})(\overline{A} + \overline{B})(\overline{B} + C)$

$\downarrow \downarrow$ $\downarrow \downarrow$ $\downarrow \downarrow$ $\downarrow \downarrow$ $\downarrow \downarrow$
 0 1 1 1 1 1 0

A \ BC	00	01	10	11
0		0	0	0
1				

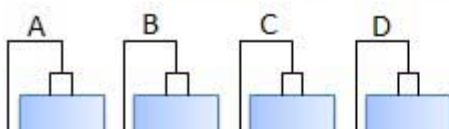
$\prod m(1,3,2)$

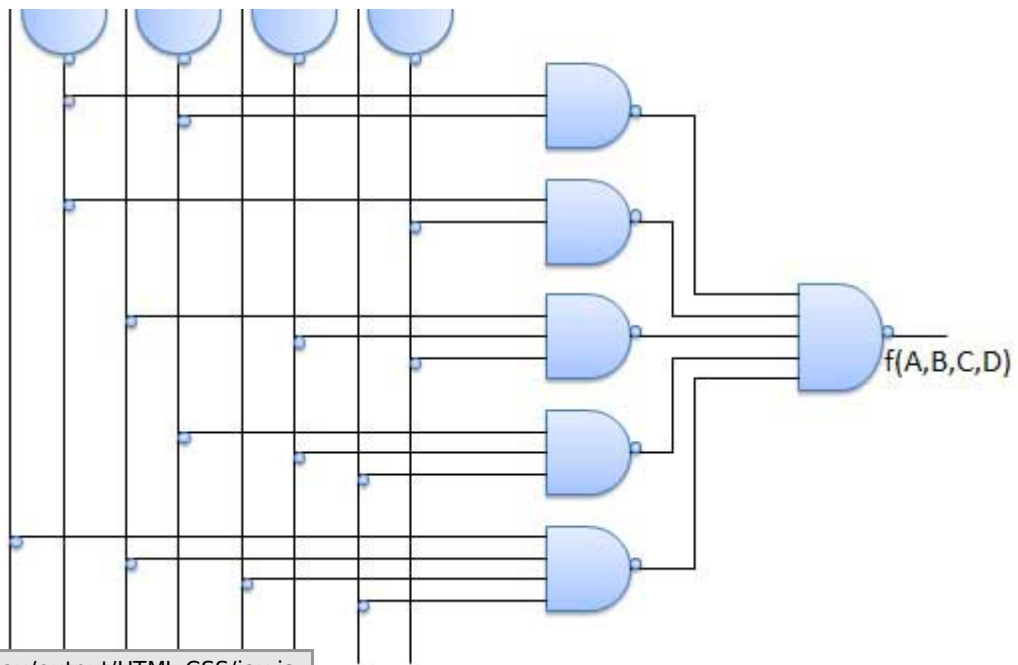
Answer : $(A + \overline{C})(A + \overline{B})$

NAND gates Realization

NAND gates can be used to simplify Boolean functions as shown in the example below.

$$F(A,B,C,D) = \overline{A}\overline{D} + ABCD + \overline{B}\overline{C}D + B\overline{C}\overline{D} + \overline{A}\overline{B}$$





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