LINE GENERATION ALGORITHM

A line connects two points. It is a basic element in graphics. To draw a line, you need two points between which you can draw a line. In the following three algorithms, we refer the one point of line as X_0, Y_0 and the second point of line as X_1, Y_1.

DDA Algorithm

Digital Differential Analyzer DDA algorithm is the simple line generation algorithm which is explained step by step here.

Step 1 − Get the input of two end points (X_0, Y_0) and (X_1, Y_1).

Step 2 − Calculate the difference between two end points.

```
dx = X1 - X0
dy = Y1 - Y0
```

Step 3 − Based on the calculated difference in step-2, you need to identify the number of steps to put pixel. If dx > dy, then you need more steps in x coordinate; otherwise in y coordinate.

```
if (dx > dy)
   Steps = absolute(dx);
else
   Steps = absolute(dy);
```

Step 4 − Calculate the increment in x coordinate and y coordinate.

```
Xincrement = dx / (float) steps;
Yincrement = dy / (float) steps;
```

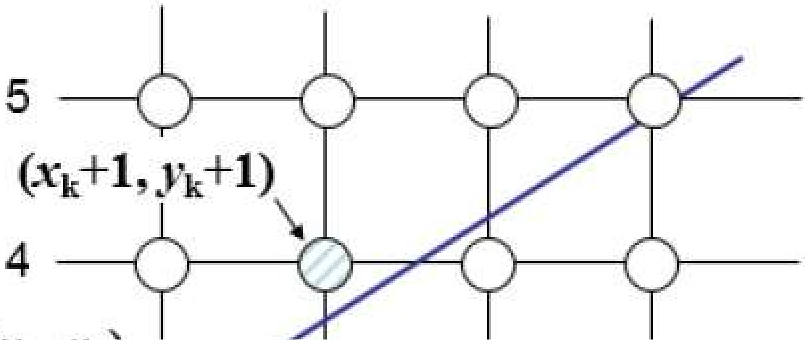
Step 5 − Put the pixel by successfully incrementing x and y coordinates accordingly and complete the drawing of the line.

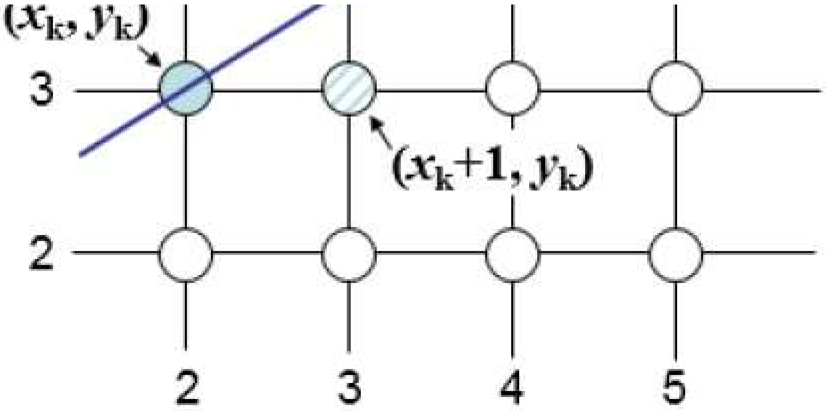
```
for(int v=0; v < Steps; v++)
{
   x = x + Xincrement;
   y = y + Yincrement;
   putpixel(x,y);
}
```

Bresenham’s Line Generation

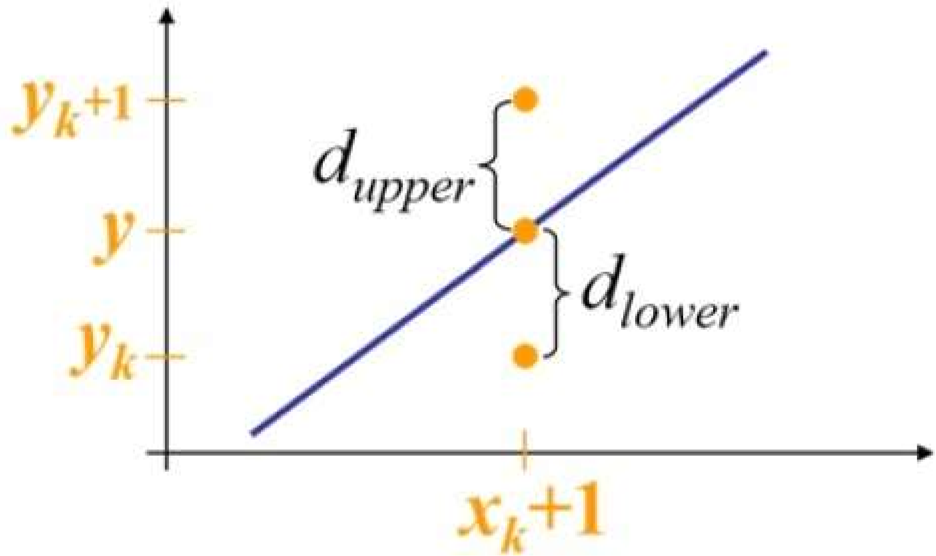
The Bresenham algorithm is another incremental scan conversion algorithm. The big advantage of this algorithm is that, it uses only integer calculations. Moving across the x axis in unit intervals and at each step choose between two different y coordinates.

For example, as shown in the following illustration, from position $2, 3$ you need to choose between $3, 3$ and $3, 4$. You would like the point that is closer to the original line.





At sample position $X_k + 1$, the vertical separations from the mathematical line are labelled as d_{upper} and d_{lower}.



From the above illustration, the y coordinate on the mathematical line at $x_k + 1$ is −

Y = mX_k + 1 + b

So, d_{upper} and d_{lower} are given as follows −

$$d_{lower} = y - y_k$$

$$= m(X_k + 1) + b - Y_k$$

and

$$d_{upper} = (y_k + 1) - y$$

$$= Y_k + 1 - m(X_k + 1) - b$$

You can use these to make a simple decision about which pixel is closer to the mathematical line. This simple decision is based on the difference between the two pixel positions.

$$d_{lower} - d_{upper} = 2m(x_k + 1) - 2y_k + 2b - 1$$

Let us substitute *m* with *dy/dx* where *dx* and *dy* are the differences between the end-points.

$$dx(d_{lower} - d_{upper}) = dx(2\frac{dy}{dx}(x_k + 1) - 2y_k + 2b - 1)$$

$$= 2dy.\, x_k - 2dx.\, y_k + 2dy + 2dx(2b - 1)$$

$$= 2dy.\, x_k - 2dx.\, y_k + C$$

So, a decision parameter P_k for the kth step along a line is given by −

$$p_k = dx(d_{lower} - d_{upper})$$

$$= 2dy.\, x_k - 2dx.\, y_k + C$$

The sign of the decision parameter P_k is the same as that of $d_{lower} - d_{upper}$.

If p_k is negative, then choose the lower pixel, otherwise choose the upper pixel.

Remember, the coordinate changes occur along the x axis in unit steps, so you can do everything with integer calculations. At step k+1, the decision parameter is given as −

$$p_{k+1} = 2dy.\, x_{k+1} - 2dx.\, y_{k+1} + C$$

Subtracting p_k from this we get −

$$p_{k+1} - p_k = 2dy(x_{k+1} - x_k) - 2dx(y_{k+1} - y_k)$$

But, x_{k+1} is the same as x_{k+1}. So −

$$p_{k+1} = p_k + 2dy - 2dx(y_{k+1} - y_k)$$

Where, $Y_{k+1} – Y_k$ is either 0 or 1 depending on the sign of P_k.

The first decision parameter p_0 is evaluated at (x_0, y_0) is given as −

$$p_0 = 2dy - dx$$

Now, keeping in mind all the above points and calculations, here is the Bresenham algorithm for slope m < 1 −

Step 1 − Input the two end-points of line, storing the left end-point in (x_0, y_0).

Step 2 − Plot the point (x_0, y_0).

Step 3 − Calculate the constants dx, dy, 2dy, and $2dy – 2dx$ and get the first value for the decision parameter as −

$$p_0 = 2dy - dx$$

Step 4 − At each X_k along the line, starting at k = 0, perform the following test −

If p_k < 0, the next point to plot is $(x_k + 1, y_k)$ and

$$p_{k+1} = p_k + 2dy$$

Otherwise,

$$p_{k+1} = p_k + 2dy - 2dx$$

Step 5 − Repeat step 4 $dx – 1$ times.

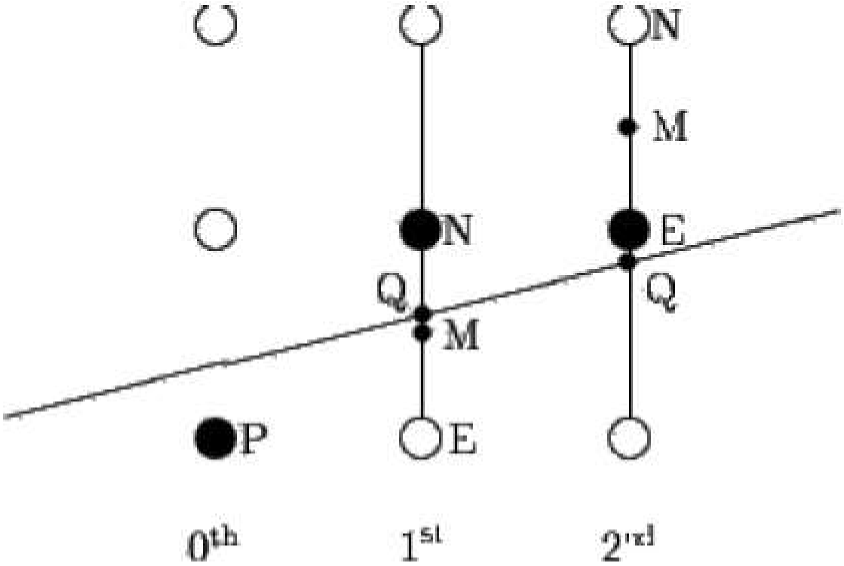
For m > 1, find out whether you need to increment x while incrementing y each time.

After solving, the equation for decision parameter P_k will be very similar, just the x and y in the equation gets interchanged.

Mid-Point Algorithm

Mid-point algorithm is due to Bresenham which was modified by Pitteway and Van Aken. Assume that you have already put the point P at x, y coordinate and the slope of the line is $0 \leq k \leq 1$ as shown in the following illustration.

Now you need to decide whether to put the next point at E or N. This can be chosen by identifying the intersection point Q closest to the point N or E. If the intersection point Q is closest to the point N then N is considered as the next point; otherwise E.



To determine that, first calculate the mid-point $M_{x+1,\, y+½}$. If the intersection point Q of the line with the vertical line connecting E and N is below M, then take E as the next point; otherwise take N as the next point.

In order to check this, we need to consider the implicit equation −

$$F_{x,y} = mx + b - y$$

For positive m at any given X,

- If y is on the line, then $F_{x,y} = 0$
- If y is above the line, then $F_{x,y} < 0$
- If y is below the line, then $F_{x,y} > 0$