PUSHDOWN AUTOMATA INTRODUCTION

Basic Structure of PDA

A pushdown automaton is a way to implement a context-free grammar in a similar way we design DFA for a regular grammar. A DFA can remember a finite amount of information, but a PDA can remember an infinite amount of information.

Basically a pushdown automaton is −

"Finite state machine" + "a stack"

A pushdown automaton has three components −

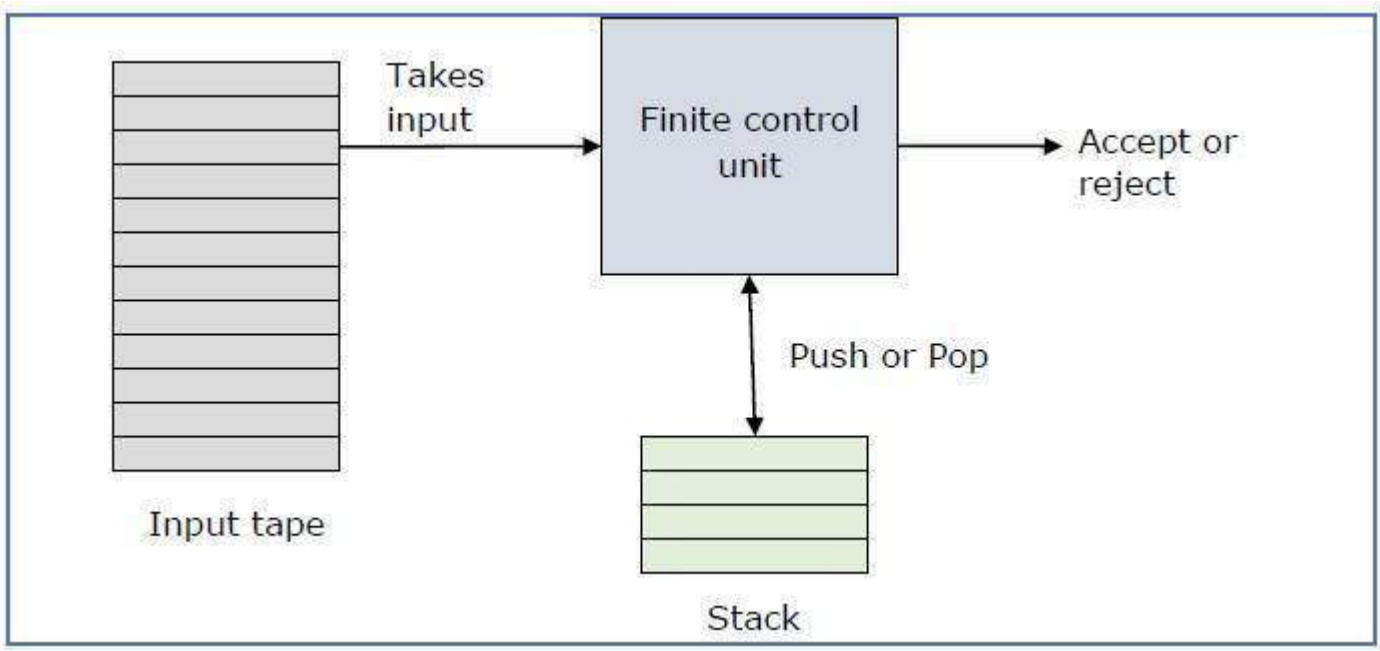
- an input tape,
- a control unit, and
- a stack with infinite size.

The stack head scans the top symbol of the stack.

A stack does two operations −

- **Push** − a new symbol is added at the top.
- **Pop** − the top symbol is read and removed.

A PDA may or may not read an input symbol, but it has to read the top of the stack in every transition.

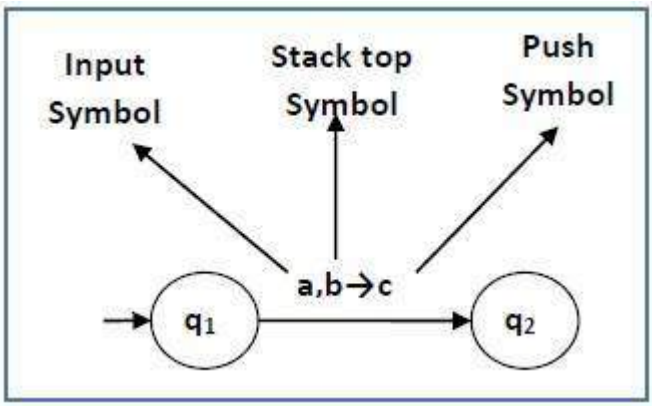


A PDA can be formally described as a 7-tuple $(Q, \sum, S, \delta, q_0, I, F)$ −

- **Q** is the finite number of states
- **∑** is input alphabet
- **S** is stack symbols
- **δ** is the transition function − $Q \times \sum \cup \varepsilon \times S \times Q \times S^*$

- **q_0** is the initial state ($q_0 \in Q$)
- **I** is the initial stack top symbol $I \in S$
- **F** is a set of accepting states $F \in Q$

The following diagram shows a transition in a PDA from a state q_1 to state q_2, labeled as a,b → c −



This means at state **q_1**, if we encounter an input string **'a'** and top symbol of the stack is **'b'**, then we pop **'b'**, push **'c'** on top of the stack and move to state **q_2**.

Terminologies Related to PDA

Instantaneous Description

The instantaneous description ID of a PDA is represented by a triplet q, w, s where

- **q** is the state
- **w** is unconsumed input
- **s** is the stack contents

Turnstile Notation

The "turnstile" notation is used for connecting pairs of ID's that represent one or many moves of a PDA. The process of transition is denoted by the turnstile symbol "⊢".

Consider a PDA (Q, ∑, S, δ, q_0, I, F). A transition can be mathematically represented by the following turnstile notation −

```
(p, aw, Tβ) ⊢ (q, w, αb)
```

This implies that while taking a transition from state **p** to state **q**, the input symbol **'a'** is consumed, and the top of the stack **'T'** is replaced by a new string **'α'**.

Note − If we want zero or more moves of a PDA, we have to use the symbol **($\vdash^*$)** for it.