DFA MINIMIZATION

DFA Minimization using Myphill-Nerode Theorem

Algorithm

Input: DFA

Output: Minimized DFA

Step 1 Draw a table for all pairs of states (Q_i, Q_j) not necessarily connected directly [All are unmarked initially]

Step 2 Consider every state pair (Q_i, Q_j) in the DFA where $Q_i \in F$ and $Q_j \notin F$ or vice versa and mark them. [Here F is the set of final states].

Step 3

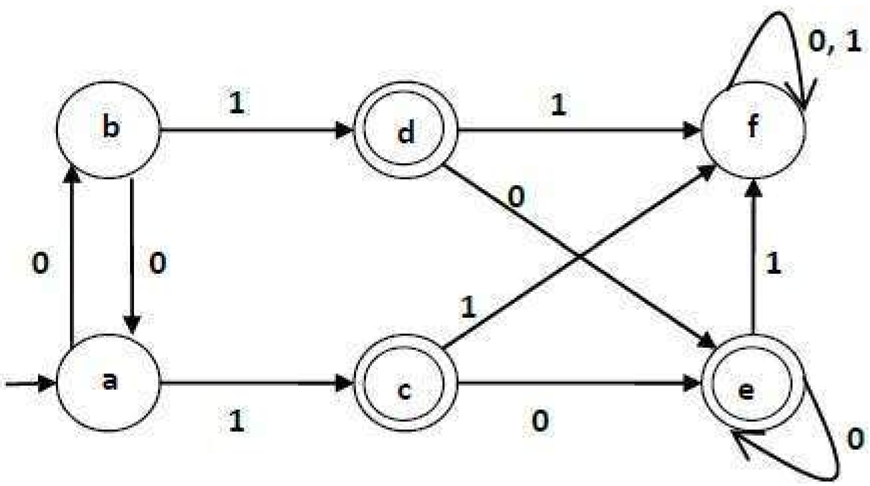
Repeat this step until we cannot mark anymore states −

If there is an unmarked pair (Q_i, Q_j), mark it if the pair $\{\delta(Q_i, A), \delta(Q_i, A)\}$ is marked for some input alphabet.

Step 4 Combine all the unmarked pair (Q_i, Q_j) and make them a single state in the reduced DFA.

Example

Let us use above algorithm to minimize the DFA shown below.



Step 1 − We draw a table for all pair of states.

	a	b	c	d	e	f
a						
b						
c						
d						
e						

f						

Step 2 − We mark the state pairs −

	a	**b**	**c**	**d**	**e**	**f**
a						
b						
c	✔	✔				
d	✔	✔				
e	✔	✔				
f			✔	✔	✔	

Step 3 − We will try to mark the state pairs, with green colored check mark, transitively. If we input 1 to state 'a' and 'f', it will go to state 'c' and 'f' respectively. c, f is already marked, hence we will mark pair a, f. Now, we input 1 to state 'b' and 'f'; it will go to state 'd' and 'f' respectively. d, f is already marked, hence we will mark pair b, f.

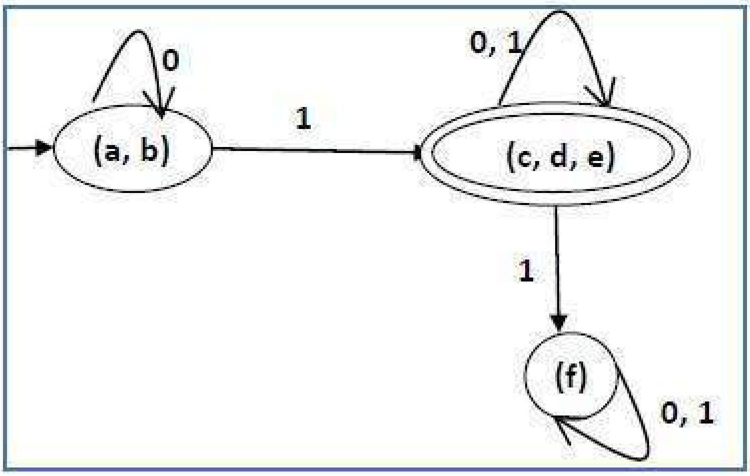
	a	**b**	**c**	**d**	**e**	**f**
a						
b						
c	✔	✔				
d	✔	✔				
e	✔	✔				
f	✔	✔	✔	✔	✔	

After step 3, we have got state combinations {a, b} {c, d} {c, e} {d, e} that are unmarked.

We can recombine {c, d} {c, e} {d, e} into {c, d, e}

Hence we got two combined states as − {a, b} and {c, d, e}

So the final minimized DFA will contain three states {f}, {a, b} and {c, d, e}



DFA Minimization using Equivalence Theorem

If X and Y are two states in a DFA, we can combine these two states into {X, Y} if they are not distinguishable. Two states are distinguishable, if there is at least one string S, such that one of δ X, S and δ Y, S is accepting and another is not accepting. Hence, a DFA is minimal if and only if all the states are distinguishable.

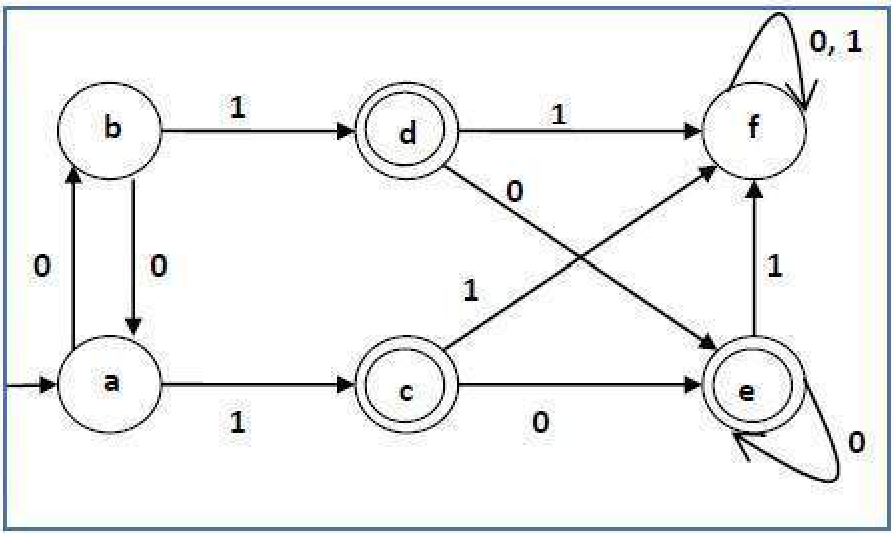
Algorithm

Step 1	All the states **Q** are divided in two partitions − **final states** and **non-final states** and are denoted by $\mathbf{P_0}$. All the states in a partition are 0^{th} equivalent. Take a counter **k** and initialize it with 0.
Step 2	Increment k by 1. For each partition in P_k, divide the states in P_k into two partitions if they are k-distinguishable. Two states within this partition X and Y are k-distinguishable if there is an input **S** such that **δ**X, S and **δ**Y, S are $k-1$-distinguishable.
Step 3	If $P_k \neq P_{k-1}$, repeat Step 2, otherwise go to Step 4.
Step 4	Combine k^{th} equivalent sets and make them the new states of the reduced DFA.

Example

Let us consider the following DFA −

q	δ$q, 0$	δ$q, 1$
a	b	c
b	a	d
c	e	f
d	e	f
e	e	f
f	f	f



Let us apply above algorithm to the above DFA −

- $P_0 = \{c, d, e, a, b, f\}$

- $P_1 = \{c, d, e, a, b, f\}$
- $P_2 = \{c, d, e, a, b, f\}$

Hence, $P_1 = P_2$.

There are three states in the reduced DFA. The reduced DFA is as follows −

The State table of DFA is as follows −

Q	$\delta q, 0$	$\delta q, 1$
a, b	a, b	c, d, e
c, d, e	c, d, e	f
f	f	f

Its graphical representation would be as follows −